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Examination of the Distribution of the Number of Component-Damage States

Lawrence D. Losie

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In live-fire testing, munitions are fired at armored vehicles. Results from this type of testing are critical components that were damaged in the encounter between the munition and the armored vehicle. Before testing begins, predictions are made by using a stochastic vulnerability estimation code. For every sampling trial, the code produces a vector that identifies a specific combination of critical components rendered nonfunctional in the encounter. Thus, each vector identifies a hypothetical test outcome. Experimental results can then be compared with predictions. As part of a consistency examination, observed component damage in a test firing can be compared with vectors of predicted damage. In some cases, a match may not be found. Such a discrepancy may be caused by an insufficient number of samples selected. By making appropriate assumptions, this sampling problem then surrenders to mathematical investigation. Illustrative examples are presented that demonstrate the nature of the sampling deficiency. Even though the examples represent idealized situations, they do provide insights to the sampling problem in the vulnerability code.

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1. Introduction

Live-fire testing makes evident the lethality of a munition or the vulnerability of a target. From previous and current live-fire test programs, targets have been armored vehicles. For munitions, live-fire testing exhibits munition lethality by showing the ability of a munition to destroy armored vehicles. For armored vehicles, live-fire testing exhibits vehicle vulnerability by showing how susceptible a vehicle is to damage when struck by a munition. Of all components in a vehicle, a certain subset is considered critical. Tactical functions will be degraded when an encounter between a munition and an armored vehicle destroys critical components or, at the very least, renders them nonfunctional or degraded. In each firing, critical components that were rendered nonfunctional are the experimental outcome from a test shot.

Before testing commences, predictions are made by using the vulnerability code, *SQuASH* (Stochastic Quantitative Assessment of System Hierarchies). *SQuASH* simulates the interaction between a munition and an armored vehicle, and varies the capabilities of resulting damage mechanisms primarily penetration and spall affecting critical components. *SQuASH* also conducts sampling to produce vectors of component damage. Each element of the vector, representing some critical component, identifies whether it has survived or has been rendered nonfunctional by the action of damage mechanisms. Every vector represents some state of damage. So, any state or vector identifies a specific combination of damaged components. For each shot, generally 1000 trials are performed to produce distinct states or vectors of component damage along with their observed likelihood of occurrence.

From *SQuASH* predictions, each component-damage vector represents a hypothetical experimental outcome. Thus, test results can be compared with predictions. As part of a consistency examination, test results are identified in the list of predicted component-damage vectors. For some shots, no match may occur between test results and predictions. In such a situation, there may be deficiencies in modeling. On the other hand, if modeling is accurate, an insufficient number of samples for a comparison may have been selected in the predictions. If too few samples have been taken, a question may be raised on how many samples are needed to ensure that observing all possible component-damage states becomes likely.

When a munition perforates the armored shell of a vehicle, different damage mechanisms produced by this interaction may render various components nonfunctional. Consider the case where independence can be assumed between every possible pair of critical components. In this case, probabilities for component loss are also assumed to be precisely known. Then, the sampling question surrenders to mathematical investigation. For such an idealized case, this report demonstrates the nature of the sampling problem. An illumination of the problem is presented. However, no solutions are provided. Such solutions are beyond the scope of this report.

A problem related to games of chance was proposed to Pascal by the Chevalier de Méré. This problem became the seed for letter correspondence between Pascal and Fermat that eventually laid the foundation for the mathematical theory of probability.* Throughout its

*E. T. Bell, Men of Mathematics, 1937, pgs. 85-89.

development, gambling and probability theory have been intertwined. In a similar vein, this sampling question is related to a gambling problem that during the mid-1980s attracted the attention of this author.

2. An Anecdotal History

In 1985, this author shared an office with three technicians. At that time, there was more than usual interest in the Maryland *LOTTO* game, since no one had won the lottery in several weeks. Hence, the jackpot had become excessively large. One day during lunch, our branch chief came into our office and spoke primarily to two of the technicians. This author's interest was diverted from the then current efforts to what our branch chief was saying. He had gathered inning numbers from the previous 14 weekly drawings. Tabulation of these data, which he gave to the two technicians but which has since disappeared, listed the drawn *LOTTO* numbers from 1 through 40, inclusive, with their corresponding frequencies of occurrence. An interesting observation, which our branch chief pointed out, was that there were six numbers that had not been drawn. So, he told the technicians that they should play those six numbers in the next *LOTTO* game: a typical gamblers folly! This author does not believe that our branch chief actually thought those six numbers would be drawn, but rather he told them that to drive both technicians *nuts* about a *sure* winner. (On the other hand, one never really knows about branch chiefs!) By the way, only one or maybe two of those numbers that had not been chosen in the last 14 games were selected in the next drawing.

Although not a gambler and, hence, not a player of the Maryland *LOTTO*, this author was still interested in this frequency tabulation due to a personal inclination towards numbers and things mathematical. After the branch chief left our office, the technicians and this author examined the frequencies. For some reason, which has since been forgotten, one of the technicians had to leave the office and, after his departure, this author suggested to the other technician that the fairness of the lottery could be examined by applying a statistical procedure to these data. In a single *LOTTO* game, 6 numbered balls are drawn from 40 balls without replacement. After each game, the selected balls are replaced. Over many games, the likelihood of drawing a specific numbered ball should be the same for all balls. Selecting the Chi-Square test was the obvious choice to examine whether each of the 40 values had the same probability of being drawn, as opposed to some of them having different probabilities. Expected frequency per lottery number was 2.1, because there was a sample of size 84 (i.e., 6 numbers per drawing and 14 weekly drawings) and 40 ordinal values. In applying the Chi-Square test, a minimal expected frequency of five is usually required to have confidence in the outcome of this statistical procedure. To obtain this confidence, the 40 ordinal values were grouped into 10 classes of length 4 (i.e., ordinal values 1 through 4 were placed in one class, 5 through 8 were placed in the next, etc.). The frequency of each class became the sum of frequencies of its corresponding four members. Now, expected class frequency became 8.4. The result from applying the Chi-Square test supported a claim of equal likelihood at the 5% level of significance. Thus, fairness of the lottery was supported by this statistical measure.

Aside from examining whether the distribution of drawn *LOTTO* numbers was fair, another aspect of these data was intriguing. A question arose in this author's mind that approximated the LOTTO situation: "Is it reasonable not to observe 6 distinct values when sampling 40 ordinal values with replacement 84 times?," as well as a more general question: "What is the distribution of distinct values not observed when sampling m values with replacement N times?" These questions were more difficult than the question of fairness. Over a short period of time, this author was sufficiently interested in these questions to investigate the distribution of number of unobserved ordinal values.

3. Mathematical Structure

In the more general case, this gambling problem is equivalent to taking samples of size N with replacement from m lottery balls or jars or bins. It has a multinomial sampling distribution,

$$\frac{N!}{x_1!x_2!\cdots x_m!}p_1^{x_1}p_2^{x_2}\cdots p_m^{x_m} , \tag{1}$$

where both $x_1 + x_2 + \cdots + x_m = N$ and $p_1 + p_2 + \cdots + p_m = 1$. The multinomial distribution can be collapsed into any of m binomial distributions,

$$\frac{N!}{x_k!(N-x_k)!}p_k^{x_k}(1-p_k)^{N-x_k} , \tag{2}$$

for $k = 1, 2, \cdots, m$. Consider the m Bernoulli random variables T_k defined as

$$T_k = \begin{cases} 0 & x_k \neq 0 \\ 1 & x_k = 0 \end{cases} . \tag{3}$$

Calculation of means and variances for these random variables becomes easier by using the binomial distribution, expression (2), rather than the original multinomial distribution, expression (1). Bernoulli random variables, T_k, have means and variances given by

$$\begin{aligned} \mu[T_k] &= (1-p_k)^N & (4) \\ \sigma^2[T_k] &= (1-p_k)^N[1-(1-p_k)^N] . & (5) \end{aligned}$$

Denote S as the sum of the m Bernoulli random variables, i.e.,

$$S = \sum_{k=1}^{m} T_k , \tag{6}$$

then S has mean and variance given by

$$\begin{aligned} \mu[S] &= \sum_{k=1}^{m}(1-p_k)^N & (7) \\ \sigma^2[S] &= \sum_{k=1}^{m}(1-p_k)^N[1-(1-p_k)^N] + \sum_{\substack{i=1 \\ i \neq j}}^{m} \sum_{j=1}^{m} Cov[T_i, T_j] . & (8) \end{aligned}$$

Bernoulli random variables T_i and T_j, $i \neq j$, are not independent, because both the sums of x's and p's are fixed. Hence, covariance terms do not necessarily vanish.

Difficulties exist in using the multinomial distribution to determine expressions for the covariance terms. Similar to what was done in determining means and variances for the Bernoulli random variables T_k, expression (1) can be collapsed into any of $m(m-1)/2$ trinomial distributions,

$$\frac{N!}{x_i!x_j!(N-x_i-x_j)!} p_i^{x_i} p_j^{x_j} (1-p_i-p_j)^{N-x_i-x_j} . \qquad (9)$$

Covariance terms can be expressed as

$$Cov[T_i, T_j] = \sum_{l=0}^{1} \sum_{m=0}^{1} (l - \mu[T_i])(m - \mu[T_j]) Pr[T_i = l \; and \; T_j = m] . \qquad (10)$$

By using the trinomial distributions, conditions for evaluating the joint probabilities can be determined

	$T_i = 0$	$T_i = 1$
$T_j = 0$	$x_i = 1, 2, \cdots, N-1$ $x_j = 1, 2, \cdots, N-1$ $x_i + x_j \leq N$	$x_i = 0$ $x_j = 1, 2, \cdots, N$
$T_j = 1$	$x_i = 1, 2, \cdots, N$ $x_j = 0$	$x_i = 0$ $x_j = 0$

,

and upon evaluation the probabilities become

	$T_i = 0$	$T_i = 1$
$T_j = 0$	$1 + (1-p_i-p_j)^N$ $-\mu[T_i] - \mu[T_j]$	$\mu[T_i] - (1-p_i-p_j)^N$
$T_j = 1$	$\mu[T_j] - (1-p_i-p_j)^N$	$(1-p_i-p_j)^N$

.

Using the joint probabilities, expression (10) simplifies to

$$Cov[T_i, T_j] = (1 - p_i - p_j)^N - \mu[T_i]\mu[T_j] . \tag{11}$$

Thus, the variance of S, expression (8), becomes

$$\sigma^2[S] = \sum_{\substack{k=1 \\ k \neq r}}^{m} \sum_{r=1}^{m} (1 - p_k - p_r)^N - \mu[S](\mu[S] - 1) . \tag{12}$$

Recall, in the lottery problem, there was a concern about whether it was reasonable that 6 out of 40 numbers would remain unselected after 84 draws. As a test of the mathematical theory, a simulation was conducted to mimic the lottery problem. The computerized simulation involved sampling equally likely ordinal values between 1 and 40, inclusive, as representatives for lottery balls, with replacement 84 times. After sampling was completed, a count of the number of values not drawn was made. The combined sampling and counting session was then replicated 999 times for a total of 1000 sessions. These counts were then tabulated as frequencies closely resembling the distribution of number of unobserved lottery balls. Results from simulating the lottery problem are shown at Table 1. When 6 lottery balls had remained unselected after 14 weekly drawings, simulation results indicate such an outcome does appear reasonable.

TABLE 1.—*Results from Simulating the Lottery Problem*

Counts	*Frequencies*
0	6
1	18
2	56
3	158
4	201
5	209
6	175
7	108
8	51
9	15
10	3

Using the frequencies, values for central tendency and variability were calculated. In comparing estimates, values from simulation differ from their theoretical values in only the second decimal place. Magnitude of differences is well within inherent randomness of the simulation. The simulated and theoretical values are shown at Table 2.

TABLE 2.—*Simulated and Theoretical Estimates for the Lottery Problem*

	Observed	*Expected*
Average	4.832	4.769
Standard Deviation	1.769	1.735

Now, S is the count of the number of lottery balls or bins or whatever that have not been observed. Using expressions (7) and (12), some observations can be made concerning the behavior of S. When $N = 1$, $\mu[S] = m - 1$ and $\sigma^2[S] = 0$, because something will always be selected with a sample of size one, thereby leaving $m - 1$ things not observed. So, with $N = 1$, S will always have a single-valued distribution. As N increases without bound, both $\mu[S]$ and $\sigma^2[S]$ will asymptotically approach zero, because as sample size increases the chance that something will remain unobserved becomes less likely. So, as N grows large, the distribution of S will approach in the limit a single-valued distribution. Between the two extremes, S will have a mean that decreases with increasing sample size. S will be nearly constant at both small and large values for N, but will have more variability at intermediate values for N.

One would be tempted to use theoretical mean and variance in a normal approximation for the distribution of the number not observed. In general usage, the normal would not be appropriate, because the Central Limit Theorem is not applicable. Recall, S is the sum of Bernoulli random variables that are neither independent nor identically distributed in conjunction with samples taken with replacement from an infinite population.

The use of the normal for this specific lottery problem just happens to accurately mimic the distribution of counts. A measure of the strength in using the normal is given by the critical level for rejection (i.e., level at which one is forced to reject a claim of normality), and in this particular situation it becomes 0.36 as measured by the Chi-Square statistical procedure. This suggests for selected range of values for N and m or the ratio N/m that the normal may adequately approximate the distribution. On the other hand, quality of a normal approximation in this case may be just a coincidence.

4. Linkage to Live-Fire Sampling Question

Suppose in a live-fire test shot there are l components susceptible to possible loss, i.e., having component probabilities of kill, p_c's, between 0 and 1, by the action of damage mechanisms produced by an encounter between a munition and an armored vehicle. Then, there are 2^l component-damage vectors. Each component vector, under the assumption of component independence, has a state probability given by,

$$p_k = \prod_{i=1}^{l} p_{c_i}^{O_i}(1 - p_{c_i})^{1-O_i}, \tag{13}$$

for $k = 1, 2, \cdots, 2^l$ and where O_i takes on a value of 0 or 1 depending upon whe he ith component, respectively, survives or becomes nonfunctional.

Using state probabilities in expression (1) with $m = 2^l$, the N samples have a multinomial sampling distribution. Then, S is the number of distinct component-damage states not observed. If W is denoted as $2^l - S$, then W represents the number of states actually observed. So, W will have mean and variance given by

$$\mu[W] = 2^l - \mu[S] \tag{14}$$
$$\sigma^2[W] = \sigma^2[S] \tag{15}$$

in terms of S, or more explicitly as

$$\mu[W] = \sum_{k=1}^{2^l}[1 - (1 - p_k)^N] \tag{16}$$
$$\sigma^2[W] = \sum_{\substack{k=1 \\ k \neq r}}^{2^l} \sum_{r=1}^{2^l} (1 - p_k - p_r)^N - U(U - 1) \tag{17}$$

with $U = 2^l - \mu[W]$. W will be an increasing function of sample size. At $N = 1$, $\mu[W]$ will be 1, and as N increases, it will asymptotically approach 2^l. Like S, W will be nearly constant at both small and large values for N, but will have more variability at intermediate values for N.

5. Examples

Four examples that have been chosen to be simple, yet informative will now be considered. They will be presented as two pairs, so the examples can more easily be compared, as well as contrasted.

The first pair consists of two examples where they differ only in the number of components susceptible to loss. In the 2 examples, the first has 5 components while the second has 10 components. In both examples, each component has a 0.5 probability of loss (i.e., a fair coin-flip situation). There are 32 states in the first example, where each state has the same probability of occurrence, i.e., 1/32. In the second example, there are 1024 equally likely states, where each has the same state probability of 1/1024. The expected number of observed states at selected values for N is shown at Table 3. In both examples, the theoretical mean initially exhibits a nearly linear growth until it sharply slows. To obtain a similar fraction of all states, more samples are required for 10 component case than for 5 component case.

TABLE 3.—*First Pair of Examples: The Effect of Changing Only the Number of Components*

Five Components *All Component Probabilities* : 0.5 32 *States* *All State Probabilities* : 1/32			*Ten Components* *All Component Probabilities* : 0.5 1024 *States* *All State Probabilities* : 1/1024		
N	$\mu[W]$	$\mu[W]/2^l$	N	$\mu[W]$	$\mu[W]/2^l$
10	8.705	0.272	250	221.917	0.217
20	15.042	0.470	500	395.741	0.386
30	19.655	0.614	1000	638.542	0.624
50	25.458	0.796	1500	787.508	0.769
70	28.533	0.892	2000	878.904	0.858
90	30.163	0.943	3000	969.383	0.947
120	31.291	0.979	4000	1003.441	0.980
150	31.726	0.991	5000	1016.261	0.992

The third and fourth examples deal with the same number of components (five), but differ in their components' probabilities of loss. In the third example, each component has a 0.7 probability of loss while the individual component probability of loss in the fourth example is 0.9. There are the same number of component-damage states for each example. In the third example, there are 32 different states having varying probabilities of occurrence: 1 with a 0.00243 probability, 5 with 0.00567, 10 with 0.01323, 10 with 0.03087, 5 with 0.07203 and 1 with 0.16807. Similar to the third example, the fourth also has 32 different states with varying likelihoods of occurrence, but the list of probabilities differs from those in the third example: 1 with a 0.00001 probability, 5 with 0.00009, 10 with 0.00081, 10 with 0.00729, 5 with 0.06561, and 1 with 0.59049. The expected number of observed states at selected values for N is shown at Table 4 for both examples. Similar to what was exhibited for the first pair, an initial growth of the theoretical mean is also nearly linear until it drastically slows. Likewise, in achieving similar number of states, the example having a common component probability of loss of 0.9 requires many more sampling trials than for the example where the common component probability of loss is 0.7.

TABLE 4.—*Second Pair of Examples: The Effect of Changing Only the Common Component Probability of Loss*

Five Components *All Component Probabilities* : 0.7 32 *States* *Varying State Probabilities*			*Five Components* *All Component Probabilities* : 0.9 32 *States* *Varying State Probabilities*		
N	$\mu[W]$	$\mu[W]/2^l$	N	$\mu[W]$	$\mu[W]/2^l$
10	7.713	0.241	100	12.000	0.375
25	14.206	0.444	250	16.342	0.511
50	20.010	0.625	500	19.299	0.603
75	23.249	0.727	1000	21.987	0.687
100	25.307	0.791	2500	25.713	0.804
250	29.887	0.934	5000	27.687	0.865
500	31.400	0.981	10000	29.060	0.908
750	31.768	0.993	50000	31.338	0.979

In the four examples, certain behavior traits were exhibited. In all examples, almost linear growth was exhibited for a small number of samples and that growth decreases as number of samples increases. The number of samples necessary to achieve a similar proportion of the total states appears to increase in the first two examples as both the number of states increases and the likelihood of state probabilities decreases. In contrast, the amount of necessary sampling appears to increase when state probabilities become both smaller and larger in the last two examples. Some of these apparent trends are correct while others are false. Indeed, the trends are a part of the subject matter for consideration in the next section.

6. Discussion

If the examples from the previous section are arranged by number of trials required to obtain a large percentage of all states, their rankings from least to greatest would be the first example, the third example, the second example and the fourth example. When ordering the smallest state probability in each example from largest to smallest, there is a one-to-one correspondence between ordered probabilities and example rankings: 1/32 in first example, 243/100000 in third, 1/1024 in second and 1/100000 in fourth. This correspondence is not a coincidence.

A reason for this correspondence can be extracted from expression (16), which is shown below for the convenience of the reader,

$$\mu[W] = \sum_{k=1}^{2^l}[1 - (1 - p_k)^N]\,.$$

The key here is the summands. Each summand is the same as the probability of observing at least one success in N trials. When a state probability is small, more sampling trials are required before a successful occurrence becomes likely. The rate by which the mean approaches 2^l will be governed by values for the state probabilities. The asymptotic approach rate can be drastically slowed when state probabilities are small. So, states having the smallest probabilities of occurrence will be the driving force.

For instance, in the first two examples, each state has the same likelihood of occurrence — in the 5 component case it is 1/32 and in the 10 component case it is 1/1024. When taking samples, every state has the same chance of being drawn whether it has or has not been previously selected. Consider the situation where sampling continues until all states but one have been drawn. In the five-component case, further draws become equivalent to Bernoulli sampling where drawing the unselected state has a probability of 1/32 and drawing of previously selected states has a probability of 31/32. Similarly, the 10 component case is equivalent to repeated Bernoulli sampling with a probability of 1/1024 for drawing the unselected state and a probability of 1023/1024 for drawing previously selected states. Drawing the unselected state will generally be more likely for the case with five components. Hence, more effort must be expended in the 10 component case than in the 5 component case to acquire the last unselected state.

The apparent influence that the number of involved components has on sampling is not exactly correct from a strict perspective. The number does have influence, which can be enormous, but rather its influence is exhibited indirectly. From the multinomial sampling distribution, expression (1), state probabilities must sum to unity. For each additional component, the number of states doubles. Due to the fixed value for the summation of state probabilities, the likelihood of occurrence for any state decreases as number of states increases. Hence, the effort in sampling a majority of states or all states will increase because of smaller state probabilities that is caused by an increased number of components.

In the last two examples, which had the same number of components but different likelihood of individual component loss, the number of trials needed to attain a similar proportion of all states was much less in the third example where each component has the same loss probability of 0.7 than in the fourth example where the common component probability was 0.9. The range of state probabilities in the third example was within the range of values in the fourth example. There was a state having a larger probability in the fourth example that initially appeared to have influence on the expected number of trials. This appearance of influence is really false, because states having larger probabilities will generally be drawn more frequently. As in the other examples, the rate by which the mean asymptotically approaches

2^l will be driven by states having smaller probabilities. The primary cause will be the state or states having the smallest likelihood of occurrence.

7. Conclusions

In making predictions for live-fire testing, a question can be raised on how many samples must be taken so that observing all component-damage states is likely. This sampling question was investigated to demonstrate the nature of the problem involved in making predictions. As stated in the introduction, no solutions are provided at this time for the prediction code, *SQuASH*, just some insights to the sampling problem are presented.

By examining some simple examples, the nature of the sampling problem was demonstrated. The examples represented idealized situations where both probability of loss for components is known and independence exists between every pair of components. Although not presented in this report, calculation of the expected number of states observed could easily be extended to situations where independence is not present, provided that the structure of component dependencies is completely specified. Furthermore, a careful reader will note that knowledge of the probability of loss for components and component independence or dependence are not absolutely necessary. What is required is just the knowledge of the number of states and precise value of the probability of occurrence for each state.

Making *SQuASH* predictions differs from idealized situations in several ways. Neither component probabilities of loss nor component-damage state probabilities will be known. Such probabilities are estimated from sampling. Total number of states will be known only when components are independent. With dependencies between components, the total number of states having non-zero probabilities will be unknown. Predictions involving states having low probabilities, whether components are independent or not, will generally require much more, if not excessive, sampling so that all states can be selected.

In the predictive code, state probabilities, along with the number of damage states with non-zero probability of occurrence, are not known before any samples are taken. Estimates for such quantities can be obtained only after sampling has been completed. The estimated quantities are precise only if a sufficient number of samples have been selected. A desirable upgrade to the predictive code would be some method of accurately measuring these quantities as sampling is taking place.

Aside from quantities not being completely specified, there is another issue: If all hypothetical outcomes are not generated, there is a risk of erroneously deducing that for some shot *SQuASH* is invalid or has serious deficiencies when in reality an experimental outcome of low probability was actually observed. To avoid making such a spurious judgment, all possible damage states must be generated. However, the effort involved in attempting to generate all possible states may not be worth the ensuing cost. This risk versus effort could be another area for future research.

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